# Nuclear model effects in neutrino-nucleus quasielastic scattering

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Nuclear model effects in neutrino-nucleus quasielastic scattering are studied within the distorted wave impulse approximation, using a relativistic shell model to describe the nucleus, and comparing it with the relativistic Fermi gas. Both charged-current and neutral-current processes are considered and, for the neutral-current case, the uncertainties that nuclear effects may introduce in measurements of the axial strange form-factor of the nucleon are investigated.

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## 1. Introduction

The interest in neutrino-nucleus scattering physics has rapidly increased in the past few years, triggered by the rich experimental program aimed at studying neutrino properties which is currently under development. The proper interpretation of future new data requires an accurate treatment of neutrino-nucleus interactions, in order to minimize systematic uncertainties due to nuclear modeling. Additionally, the new neutrino facilities provide an opportunity to measure nucleon properties, such as the nucleon weak form-factors. Such measurements will be obtained from neutrino-nucleus scattering data and, again, an accurate treatment of nuclear model effects will be needed for a proper interpretation of the results in terms of single nucleons. Initially a Fermi gas description of the target nucleus was considered to be appropriate, but several studies have shown that this description needs to be improved. To this purpose an important guide is provided by electron-nucleus scattering, for which both theory and experiment have reached a very high degree of sophistication.

In this contribution we address the problem of nuclear model effects in neutrino-nucleus quasielastic (QE) scattering, which has been studied by many authors in the past few years [1, 2, 3, 4, 5, 6]. We compare relativistic

Fermi gas (RFG) calculations of cross sections with results obtained within a relativistic shell model (RSM), originally developed for describing $(e, e'N)$ reactions, considering both charged-current (CC) and neutral current (NC) processes. The former are relevant for the physics of neutrino detectors, while the latter can be used for measuring the nucleon axial strange form-factor. In this case, we also consider the ratio of NC over CC cross sections.

## 2. Formalism

Let us consider the QE processes

$$\nu_\mu + A \rightarrow \mu^- + p + (A-1) \tag{1}$$

$$\nu_\mu + A \rightarrow \nu_\mu + p + (A-1)\,, \tag{2}$$

where a neutrino of four momentum $K = (\epsilon, \mathbf{k})$ interacts with a nucleus A, producing a final state in which a lepton of momentum $K' = (\epsilon', \mathbf{k}')$, an emitted nucleon of momentum $P_N = (E_N, \mathbf{p}_N)$ and the (unobserved) residual nucleus $(A-1)$ are present. Following standard procedures the exclusive cross section for these processes can be written as a contraction of a leptonic and a hadronic tensor

$$\frac{d\sigma}{d^3k' d^3p_N} \propto \eta_{\mu\nu} W^{\mu\nu}\,. \tag{3}$$

The exclusive cross section is then integrated over the momentum of the final lepton and/or nucleon in order to obtain the observables of interest. In the following, for the CC processes (1) we integrate over the emitted proton and consider the inclusive cross section $(d\sigma/dT_\mu)$, where $T_\mu$ is the outgoing lepton kinetic energy. For the NC processes (2) we integrate over the undetectable outgoing neutrino and consider $(d\sigma/dT_N)$, $T_N$ being the kinetic energy of the emitted nucleon. The leptonic tensor in Eq. (3) is given by

$$\eta_{\mu\nu} = K_\mu K'_\nu - g_{\mu\nu} K \cdot K' + K'_\mu K_\nu - i\epsilon_{\mu\nu\rho\sigma} K^\rho K'^\sigma\,. \tag{4}$$

The hadronic tensor $W^{\mu\nu}$ is given in general as a bilinear combination of matrix elements of the full nuclear weak current, taken between the target nucleus ground state and a final state written as a product of the residual nucleus (A-1) times the outgoing nucleon scattering state $\phi_N$, and summed over all the states of the residual system:

$$W^{\mu\nu} = \sum_{(A-1)} \langle A{-}1, \phi_N | \hat{J}^\mu(\mathbf{q}) | A\rangle \langle A{-}1, \phi_N | \hat{J}^\nu(\mathbf{q}) | A\rangle^* \delta(E_A{+}\omega{-}E_{A-1}{-}E_N)\,. \tag{5}$$

We calculate $W^{\mu\nu}$ within the impulse approximation, assuming (i) that the incident neutrino interacts with only one nucleon which is then emitted, while the remaining (A-1) nucleons in the target are spectators, (ii) that the nuclear current is the sum of single nucleon currents, and (iii) that the target and residual nuclei can be adequately described within an independent particle model. Under these assumptions the matrix elements contributing to $W^{\mu\nu}$ are greatly simplified and reduce to single nucleon matrix elements

$$\langle A-1, \phi_N | \hat{J}^\mu | A \rangle \rightarrow \langle \phi_N | \hat{J}^\mu_{S.N.} | \psi_B \rangle \, . \tag{6}$$

Here $\hat{J}^\mu_{S.N.}$ is the single nucleon current operator, which we parametrize in a standard way in term of vector and axial weak form-factors (see for example [7]) and $\psi_B$ and $\phi_N$ are the wave functions describing the initial bound nucleon and the outgoing nucleon, respectively. In the results presented in next section, these wave functions are calculated using the "Madrid-Seville" model [8], originally developed for describing exclusive electron scattering reactions and later employed extensively also for studying neutrino scattering [1, 2, 3, 4, 5].

In this model the bound nucleon wave functions are obtained as solutions of a Dirac equation derived within a relativistic mean field approximation from a Lagrangian containing $\sigma$, $\omega$ and $\rho$ mesons. Several descriptions are possible for the outgoing nucleon wave functions. In the simplest approach, final state interactions (FSI) effects are neglected and $\phi_N$ is given by a simple plane wave Dirac spinor (plane wave impulse approximation, PWIA). In a more realistic approach $\phi_N$ is obtained as the solution of a Dirac equation containing a phenomenological relativistic optical potential (ROP), obtained from fits of elastic proton-nucleus scattering data. Such potential has a real part, describing the rescattering of the emitted nucleon, and an imaginary part, taking into account the possibility that it is absorbed into unobserved inelastic channels. This description (referred to as ROP in the following) is appropriate for the calculation of cross sections ($d\sigma/dT_N$), where the final nucleon is assumed to be detected. On the other hand, when considering inclusive cross sections ($d\sigma/dT_\mu$) a selection of the single-nucleon knockout channel cannot be made, and the contribution from the inelastic channels should be retained [1, 4]. Within our approach, a simple way to do so is to consider the outgoing nucleon wave functions obtained by setting to zero the imaginary part of the ROP (real-ROP approach), thus taking into account the conservation of the incident flux. Another possibility is to consider for $\phi_N$ the solutions in the continuum of the same relativistic mean field (RMF) equation used to obtain the nucleon bound states. Recent studies of scaling properties of inclusive CC neutrino-nucleus QE scattering seem to favor the RMF approach [4].

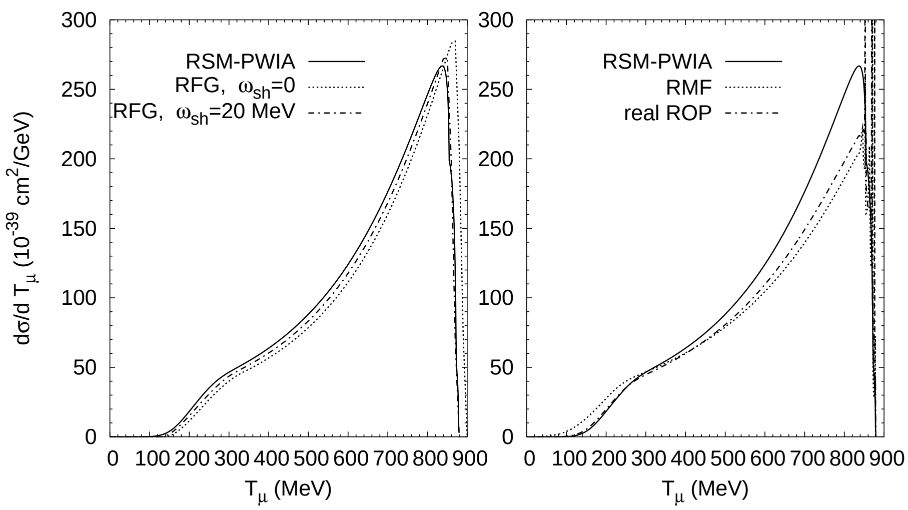


Fig. 1. Inclusive cross sections $d\sigma/dT_\mu$ for the CC processes (1) on $^{16}O$ versus the outgoing muon kinetic energy $T_\mu$ for $E_\nu = 1$ GeV. The left panel shows results in PWIA and the right panel shows the effects of FSI. RFG cross sections are calculated for Fermi momentum $k_F = 216$ MeV.

We then compare our results with those obtained within the RFG model, (see for example [2]), including in the latter also a phenomenological energy shift ($\omega_{sh}$), which is introduced in studies of inclusive electron scattering in order to get the correct position of the QE peak. In light of current/possible experiments, we focus here on incident energy $E_\nu = 1$ GeV and on oxygen and carbon targets.

## 3. Results

The differential CC cross sections ($d\sigma/dT_\mu$) for the QE scattering of muon neutrinos on $^{16}O$ are displayed in Fig. 1, as a function of the outgoing muon kinetic energy $T_\mu$ [1]. In the left panel FSI effects are neglected and the RFG results are compared with the RSM-PWIA model. We observe that, when the RFG energy shift $\omega_{sh}$ is taken into account, the differences between RFG and RSM-PWIA are very small. The right panel of the figure illustrates the effects of FSI within the RSM, by comparing the RSM-PWIA curve (solid line, the same as in the left panel) with the curves obtained using the real-ROP and RMF approaches outlined above. The sharp peak structure observed at large $T_\mu$ (small energy transfer) is typical when real

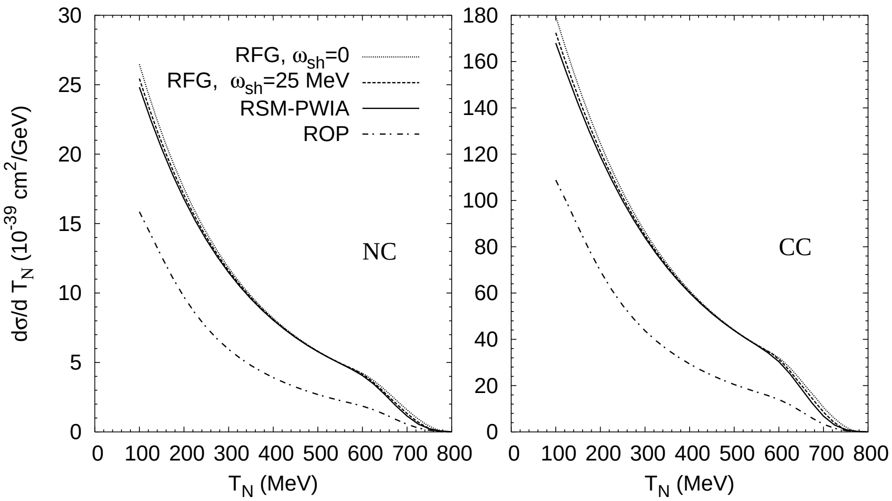


Fig. 2. Exclusive cross sections $d\sigma/dT_N$ for the NC processes (2) (left panel) and for the CC processes (1) (right panel). The target nucleus is $^{12}C$ and the neutrino energy is $E_\nu = 1$ GeV. RFG cross sections are calculated for Fermi momentum $k_F = 225$ MeV.

potentials are used in the description of the outgoing nucleon. The real-ROP and RMF curves, very close to each other, show that FSI produce a reduction of the cross section of about 10-15%, which is not too large but may have non-negligible effects in the data analysis of experiments measuring neutrino properties.

Let us now consider the differential cross section ($d\sigma/dT_N$) for $\nu_\mu$ induced proton knockout [2]. Besides NC processes (left panel), we also consider CC cross sections (right panel), which will be later used to construct ratios of cross sections. We see that, when FSI are neglected, the RSM and RFG give results that almost coincide. The inclusion of FSI, which in this case is treated using the full complex ROP, produces an important reduction of the cross section ($\simeq 50\%$). The effects are very similar for NC and CC processes.

It is clear that such big effects may have a very strong impact on the use of separate NC cross sections for measurements of the axial strange form-factor $G_A^s$. This is illustrated in the left panel of Fig. 3, where the NC cross section integrated over $T_N$ is plotted as a function of $g_A^s \equiv G_A^s(Q^2 = 0)$. Here a simple dipole parametrization has been assumed for $G_A^s(Q^2)$, with the same cut-off mass used for the non-strange axial form-factor ($M_A =$

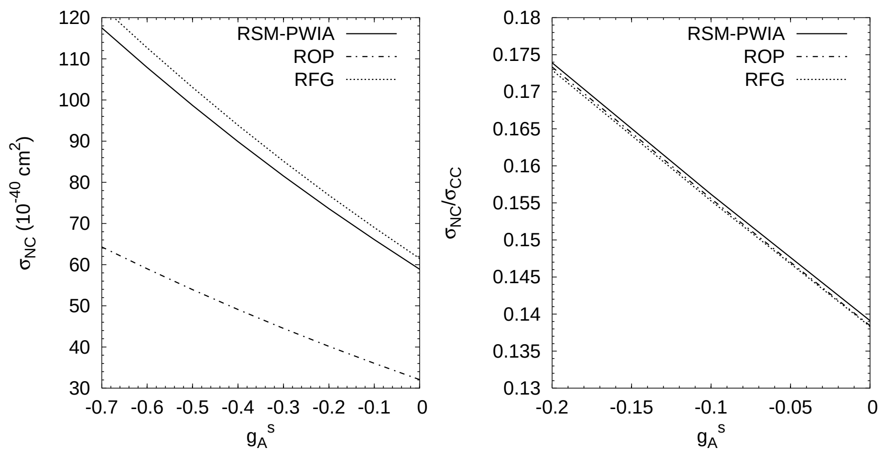


Fig. 3. Left panel: cross section for NC neutrino induced proton emission integrated over the outgoing proton kinetic energy, versus the $Q^2 = 0$ value of the axial strange from factor. The target nucleus is $^{12}C$ and the incident neutrino energy is $E_\nu = 1$ GeV. Right panel: ratio of NC and CC integrated cross sections (note the smaller range of values for $g_A^s$).

1.032 GeV).

Is is well known that, in order to extract the nucleon strange form-factors from measurements of neutrino-nucleus cross sections, nuclear model effects can be largely canceled by considering appropriate ratios of cross sections. Several observables have been considered [7, 9], but the most realistic, from the experimental point of view, seems to be the so called NC over CC ratio, obtained by dividing exclusive NC cross sections by the corresponding CC ones. The large cancellation of nuclear model uncertainties in this ratio is illustrated in the right panel of Fig. 3, where the NC/CC ratio of integrated cross sections is plotted versus $g_A^s$. We see that the very different curves shown in the left panel here almost collapse on a single line, making it possible to extract the value of $g_A^s$, provided the experimental errors are sufficiently small.

It is interesting to compare the impact of nuclear model effects on the determination of the axial strange form-factor $G_A^s$, from a measurement of the NC/CC ratio, with other possible uncertainties, in particular those due to the other form-factors of the nucleon, namely the non-strange axial form-factor $G_A$ (assumed to be measured independently in CC processes) and the vector strange form-factors (assumed to be determined from parity-violating electron scattering data [7]). This is done in Fig. 4, where the

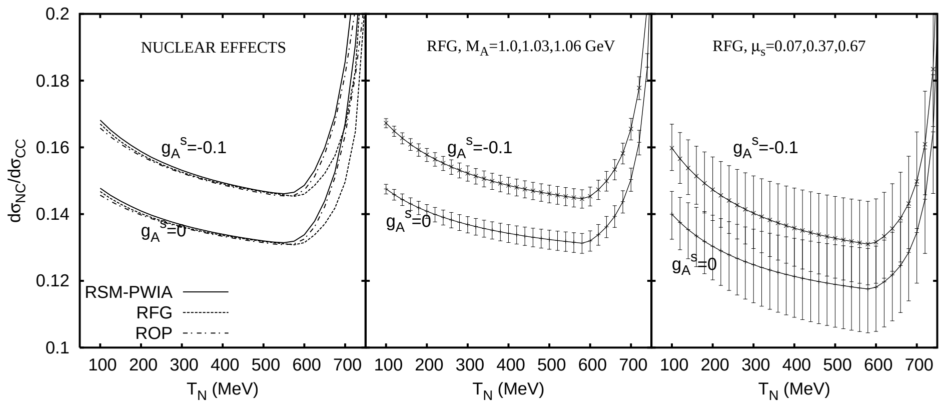


Fig. 4. NC over CC ratio of cross sections $d\sigma/dT_N$ for $\nu_\mu$ scattering on $^{12}C$ at $E_\nu = 1$ GeV, for two values of $g_A^s$ as indicated. The left panel shows the effects of using different nuclear models. The middle panel illustrates the uncertainties related to the value of the axial cutoff mass. The right panel gives an example of the possible uncertainties due to the magnetic strange form-factor $G_M^s$, here assumed to have a dipole $Q^2$ dependence (see text). In the middle and right panels we show only RFG results, which are calculated for $k_F = 225$ MeV and $\omega_{sh} = 0$.

ratio of the NC and CC differential cross sections of Fig. 2 is plotted as a function of the emitted nucleon energy $T_N$. The left panel shows the size of nuclear model uncertainties, which are very small. Again, the axial form-factors $G_A$ and $G_A^s$ are parametrized assuming a dipole dependence with the same cutoff mass $M_A$. The middle panel illustrates the effects of changing $M_A$ in the range 1.00-1.06 GeV. Finally, the right panel shows the possible uncertainties due to the magnetic strange form-factor $G_M^s$, for which we assume a dipole parametrization $G_M^S = \mu_s/(1 + Q^2/M_V^2)^2$ with $M_V^2 = 0.71$ $GeV^2$ and $\mu_s = 0.37 \pm 0.30$[10], while we assume the electric strange form-factor $G_E^s$ to be zero. At present little is known about the $Q^2$ dependence of the vector strange form-factors, and the results shown here are meant to be only an illustration of the impact that the current large errors on them may have on the NC/CC ratio. We see that the effects shown in the middle and right panels of Fig. 4 are much larger than those due to nuclear modeling.

In conclusion, we have calculated CC and NC neutrino-nucleus scattering cross sections in a relativistic shell model approach, with FSI effects taken into account within the relativistic impulse approximation. Our results for inclusive CC QE scattering show that FSI effects, although not being extremely large, can still be sizable ($\simeq 10\%$) at the relatively high incident energy $E_\nu = 1$ GeV. For exclusive NC QE processes FSI effects turn out to be very large ($\simeq 50\%$) even at high energy and may prevent a precise extraction of the axial strange form-factor of the nucleon form separate cross sections. However, these effects are almost canceled when the NC/CC ratio is considered. Within our model, nuclear model effects on ratios turn out to be much smaller than the uncertainties introduced by the single nucleon form-factors and can thus be considered well under control. Of course other uncertainties, such as those coming from non-QE contributions to the cross sections, or due to other FSI effects not taken into account in the present approach, should be carefully considered.

The results presented in this contribution have been obtained in several fruitful collaborations with W.M. Alberico, J.A. Caballero, M.C. Martínez and J.M. Udías.

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